Large scale congested transit assignment:
achievements and challenges

Michael Florian

Emma Freijinger, November 2013
Contents of presentation

- Motivation
  - Congestion aboard the vehicles
  - A large scale application
  - Congestion outside the vehicles
  - Some applications
  - Conclusions

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Some Vehicles are Truly Congested!
Sao Paulo Metro Station

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Passengers are fainting on Perth's overcrowded trains

Alyesha Anderson • The Sunday Times • August 11, 2012 9:31PM
“Sydney bus and train commuters say overcrowding is still their main public transport concern.”

“Buses in Sydney on the busiest routes are often overcrowded and do not stop for passengers, with an extraordinary 22% of people missing their service.”
Congested/Capacitated Transit Applications

- Hong Kong (2002)
- Sao Paulo (2005)
- Santiago (2010)
- San Francisco (2010)
- Mexico City (2011)
- Los Angeles (2013?)
- Rio de Janeiro (2013)
- Brisbane (2013)
- Others…
Motivation

• In many cities of the developed and developing world certain transit services are overcrowded;

• There is a need to model the congestion aboard the vehicles and the increased waiting times at stops;

• Most existing transit route choice models do not consider such congestions and capacity effects.

• Even if the transit services have sufficient capacity faster modes attract more demand and reach capacity;

• In some transit assignment models the demand overestimates the supply offered certain lines. It would be useful to determine when the demand can not be satisfied regardless of the route choice.

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This presentation will focus on applications of the methods described in:


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Congestion aboard vehicles

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Congestion aboard vehicles

- Models ‘discomfort’ function as vehicles become congested
- Increased perceived impedance through in-vehicle time
- Optimization model - unique solution

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Consider now a general transit network where each link is the segment of a transit line and has two attributes:

- travel time
- frequency

\[ t_a \]  
\[ f_a \]

\( a, b : \text{in-vehicle links} \quad (t_a, \infty) \)
\( c : \text{alight link} \quad (0, \infty) \)
\( d : \text{boarding link} \quad (0, f_a) \)

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Notation

$I$ nodes; $A$ links

$A^+_i$ outgoing links at $i$

$A^-_i$ incoming links at $i$

$A$ attractive links $(A \subseteq A)$ «strategy»

$A^+_i$ attractive outgoing links at node $i$ $(A \cap A^+_i)$

Waiting time at node $i$ for strategy $A$

$$w(A^+_i) = \frac{\alpha}{\sum_{a \in A^+_i} f_a}$$

$\alpha$ is a constant usually taken naively to be .5

Probability of leaving node $i$ on link $A$:

$$p_a(A^+_i) = \begin{cases} 
0 & \text{if } a \notin A \\
\frac{f_a}{\sum_{a' \in A^+_i} f_a'} & \text{if } a \in A 
\end{cases}$$

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The Optimal Strategy Model – basic version

\[
\min \sum_{a \in A} t_a v_a + \sum_{i \in I} \sum_{d \in D} w_i^d \\
\text{Subject to}\]

\[
v_a^d \leq f_a w_i^d, \ a \in A_i^+, \ i \in I, \ d \in D \quad (*)
\]

\[
\sum_{a \in A_i^+} v_a^d - \sum_{a \in A_i^-} v_a^d = g_i^d, \ i \in I, \ d \in D \quad \text{(flow conservation)}
\]

\[
v_a^d \geq 0, \quad a \in A, \ d \in D \quad \text{(non negativity constraints)}
\]

\[
w_i^d, \ a \in A_i^+, \ i \in I, \ d \in D \quad \text{(are the total waiting times at nodes)}
\]

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The Optimal Strategy Model – basic version

Min \([\text{total travel time + total waiting time}]\)

Subject to:

- line segment flow \(\leq\) line frequency*total waiting time
  (for all nodes and all destinations)

- flow conservation equations
  (for all nodes and all destinations)

- line segment flows \(\geq 0\) (non-negativity)
Congestion aboard vehicles – the model

- The modeling of congestion aboard the vehicles is done by associating congestion functions with the segments of transit lines to reflect the crowding effects,

- The resulting model is nonlinear and leads to a transit equilibrium model according to Wardrop’s user optimal principle stated as:

  “For all origin-destination pairs the strategies that carry flow are of minimal generalized cost and the strategies that do not carry flow are of a cost which is larger or equal to the minimal cost.”

- This leads to a convex cost optimization problem.
Modeling Congestion with Segment Crowding Functions

\[ cf(v_s) \]

\[ cf(cap_s) \]

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Crowding Function with Incremental increases (from Sydney, Australia)

\[ cf(v_s) \]

Source: PB Americas

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London Underground Crowding Functions

Crowding Curves by vehicle Type (LU)

$cf(v_s)$

Source: TfL

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The congestion aboard the transit vehicles is represented by convex increasing functions $t_a(v_a)$ for transit segments. The resulting model is

$$\min \sum_{a \in A} \int_0^{v_a} t_a(x) dx + \sum_{i \in I} \sum_{d \in D} w_i^d$$

subject to

$$v_a^d \leq f_a w_i^d, \ a \in A_i^+, \ i \in I, \ d \in D$$

$$\sum_{a \in A_i^+} v_a^d - \sum_{a \in A_i^-} v_a^d = g_i^d, \ i \in I, \ d \in D$$

$$v_a^d \geq 0, \ a \in A, \ d \in D$$
The resulting model is:

Min [sum of integral of congestion functions + total waiting time]

Subject to:  line segment flow <= line frequency*total waiting time  
              (for all nodes and all destinations)

              flow conservation equations;  
              (for all nodes and all destinations)

              non-negativity of line segment flows  
              (non-negativity)

This model does not consider that passengers can not board the first bus to arrive due to the simple fact that it may be full.

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Solution Method

- The model is solved by using an adaptation of the linear approximation method or its more efficient bi-conjugate variant;
- Each subproblem requires the computation of optimal strategies for linear cost problems;
- Convergence measures are computed just like in an equilibrium assignment of road traffic.
Convergence Measures

• The difference between the total travel time + total waiting time and the total travel time on shortest strategies is the GAP of the solution. A perfect equilibrium solution has a GAP=0.

• A well accepted stopping criterion is the Relative GAP \( RGAP = \frac{GAP}{\text{total travel time} + \text{total waiting time}} \)

• Another stopping criterion is the Normalized GAP \( NGAP = \frac{GAP}{\text{Total demand}} \)

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An Application on London

An example of a large congested transit application is: the RAILPLAN model used by TfL.

The Network size is:
  13 modes
  269 transit vehicle types
  4004 zones
  2069 transit lines
  89651 regular nodes
  202020 transit line segments
  267058 directional links

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London Underground Crowding Functions

Crowding Curves by vehicle Type (LU)

Source: TfL

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National Rail Crowding Functions

Crowding Curves by vehicle Type (NR)

Source: TfL
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Execution times

- The RAILPLAN model requires about 5 minutes per iteration (on a current generation PC) and a relative gap of $10^{-3}$ is reached in 15-20 iterations;
- The model was applied and validated against counts;
- The model was also used in an O-D matrix adjustment to fit the counts
Multithreaded speedup

- Railplan speedup
- Speedup will vary by
  - Application
  - Hardware
- Even more dramatic speedups expected on energy efficient, slightly slower CPU multicore rack servers
Assignment of Initial Matrix

Assigned Flows $R^2 = 0.952; \text{RMSE} = 3318.16; b = 1.147$

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Assignment of Adjusted Matrix

R² = 0.992; b = 1.0006; RMSE = 1144.55
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Adding Congestion Outside the Vehicle

- Riders cannot board the vehicle and have to wait for the next one. Modeled as effective line-stop-specific headway greater than the actual one.
- Similar to shadow pricing in location choices or VDF when V/C>1.

Crowding inconvenience and discomfort (demand exceeds seated capacity):
- Some riders have to stand.
- Seating passengers experience inconvenience in finding a seat and getting off the vehicle.
- Modeled as perceived weight factor on segment IVT.

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Adding Congestion Outside the Vehicle

- Models ‘discomfort’ function as vehicles become congested
  - Also adjusts waiting time at stops when passengers cannot board
  - Affects impedances
    - In-vehicle time
    - Waiting times at stops
  - Convergent iterative procedure

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Effective Frequency

• There is a need to model the limited capacity of the transit lines and the increased waiting times as the flows reach the capacity of the vehicle.

• As the transit segments become congested, the comfort level decreases and the waiting times increase.

• The mechanism used to model the increased waiting times is that of “effective frequency”.

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Effective Headway (Line & Stop Specific)

Net Capacity = Total capacity - Volume + Alight

Eff. Hdwy Factor

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• The “effective frequency” of a line is the frequency which is perceived by the transit traveler and may be less than the nominal line frequency;

• The waiting time at a stop may be modeled by using steady state queuing formulae, which take into account the residual vehicle capacity, the alightings and the boardings at stops.
A Continuous Headway Factor Function

Perceived headway = original headway * headway factor

\[
\text{original headway} \times \frac{1}{1 - \left(\frac{\text{boardings}}{\text{capacity} - (\text{volume} - \text{boardings})}\right)^{0.2}}
\]
Minimizing the gap function

- The transit assignment with capacities is solved by finding the minimum of the GAP function:

\[
\text{GAP (flow)} = \text{total travel time} + \text{total waiting time} - \text{total travel time on shortest strategies}
\]

- As for the congested transit model, a perfect equilibrium solution has a GAP=0;

- The relative gap can be used to monitor convergence just as in the congested transit model.
The Transit Assignment Model with Capacities

\[
\min \sum_{d \in D} \left[ \sum_{a \in A} t_a(v) v_a^d + \sum_{i \in N} w_i^d - \sum_{i \in N} g_i^d \tau_i^d(v) \right]
\]

\[
\sum_{a \in A_i^+} v_a^d - \sum_{a \in A_i^-} v_a^d = g_i^d, \quad i \in N, \ d \in D
\]

\[
v_a^d \leq w_i^d f_a(v), \quad a \in A_i^+, \quad i \in N, \ d \in D
\]

\[
v_a^d \geq 0, \quad a \in A, \ d \in D
\]

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A Simple Algorithm

• The solution method uses a sequence of strategies computations in a successive averaging scheme;

• The method which the deviation of a solution from the optimal solution by using the value of the objective function;

• It is also interesting to use other convergence measures such as the number of links over capacity, excess passenger volume % and the maximum segment v/c ratio.
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A Medium Size Application

- The transit services provide sufficient capacity, but
- The express lines are assigned too many trips if capacity is not considered.
Initial Uncongested Assignment
Capacity Constrained Assignment
Convergence: Relative Gap

CAPTRAS Convergence Curve
(relative gap)

iteration

relative gap

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Convergence: Excess Passenger Volume %

CAPTRAS Convergence
(excess volume %)

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Convergence: Maximum v/c Ratio

CAPTRAS Convergence
max. segment v/c

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Equilibrated Flows-Express Lines

Congestion on transit lines

- Extremely crowded
- Crowded
- No seats available
- Space available

Passenger-Hours

- 400
- 300
- 200
- 100
- 0

Lines: 14a, 15ae, 17a, 23a, 24a, 27ae, 27be, 29a, 35ae, 42ae, 49ae, 50ae, 54a, 55a, 56ae, 56be
Line 27 ae – Initial Flows

Line 27ae: king's park express

Volume
500
400
300
200
100
0

Dist. (km)
1.76 2.80 3.44 4.16 5.03 5.96 6.33 7.08 10.12 10.93 12.20 13.91 14.90 15.32 15.65 15.85 15.95

Boardings
Alightings
Thru-passengers
Line 27 ae – Equilibrated Flows

Line 27ae: king`s park express

Volume

Boardings
Alightings
Thru-passengers

Dist. (km)
Infeasible Demand

• A numerical example illustrates the results obtained when the demand is too high for the network capacity;
• The demand of the example network is increased by 50% and then assigned;
• The pedestrian mode is allowed on all the links of the road network.
• The resulting pedestrian flows on these arcs, than can not be accommodated in transit vehicles, would indicate the corridors that have insufficient capacity or that demand is over estimated.
Convergence: Relative Gap
The solution is not capacity feasible

Note that the relative gap is > 4.5%

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Convergence: Maximum Segment v/c Ratio

The solution is not capacity feasible

CAPTRAS Convergence
max. segment v/c

iteration

max. v/c ratio

0 0.5 1 1.5 2 2.5 3 3.5 4

1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70

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Santiago, Chile (2012)

• A transit assignment model on a much larger number of zones was developed by SECTRA;
• The demand was obtained by analyzing electronic fare cards;
• The results are more than satisfactory.
Emme Model – Model Construction

Trunk Routes (2011 coverage)

Regular nodes: 20,093
Stops and stations: 12,500
Centroids: 2,916
Regular links: 60,528
Connectors: 24,714

(software size and license)

Metro Lines (2011 coverage)

Trunk routes: 390 service-direction / 3,700 vehicles / 7,800 km

Feeder routes: 430 service-direction / 2,200 vehicles / 5,100 km

Source: SECTRA

Feeder Routes (2011 coverage)

Metro: 150 vehicles (trains) / 5 lines / 200 km

Gobierno de Chile | Ministerio de Transportes y Telecomunicaciones
Flows on all transit modes
Emme Model – Results with Capacity Considerations

Source: SECTRA
Emme Model – Results with Capacity Considerations

Source: SECTRA
Emme Model – Results with Capacity Considerations

Source: SECTRA
Emme Model – Results with Capacity Considerations

Source: SECTRA
Assigned vs. metro counts segment volume

R² = 0.99, STD = 438.14

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Changes in Waiting Times at Stops

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Sistema de Transporte Colectivo (STC)

- 4.1M passengers/day
- 2nd largest in N America
- 8th in world
- fare increased in 2010 to MXN 3.00 (USD 0.25)

Emme Capacitated transit assignment

Ana Fernández Olivares
Héctor Juárez Valencia
Sistema de Transporte Colectivo (STC)

Line A, La Paz to Pantitlán
Standard transit assignment
Capacitated transit assignment, iteration 4
Capacitated transit assignment, iteration 10
Capacitated transit assignment, iteration 18
Sistema de Transporte Colectivo  Mexico City

Capacitated transit assignment, iteration 24
Sistema de Transporte Colectivo (STC)

- 4.1M passengers/day
- 2\textsuperscript{nd} largest in N America
- 8\textsuperscript{th} in world
- fare increased in 2010 to MXN 3.00 (USD 0.25)

Line B, Azteca to Beunavista
Standard transit assignment
Capacitated transit assignment, iteration 2
Capacitated transit assignment, iteration 7
Some Challenges

The application of transit route choice models which take into account congestion require better data:

I. Segment congestion functions,
II. Effective frequency functions,
III. Reliable measures of capacity,
IV. And more empirical observed data for validation.
Some Challenges

- Temporal frequency based models with capacity are to be explored;
- Integration of data available on the web for itineraries and schedules.

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